The Maths Hubs and National Projects

Raising Standards in Mathematics through a Mastery Curriculum

Debbie Morgan
Director for Primary
National Projects

MASTERY

• England China Exchange Programme
• Textbook Project
What does it mean to master something?

• I know how to do it
• It becomes automatic and I don’t need to think about it- for example driving a car
• I’m really good at doing it – painting a room, or a picture
• I can show someone else how to do it.
Mastering Mathematics is more......

I don’t just need to drive the car, I need to know how it works because one day I might need to build a tractor and I need to apply my knowledge of how a car works to build a tractor.

Mathematics relies on making connections and necessitates the need to build up a network of interconnecting ideas.
Other Features of Mastery

- Deep and Sustainable Learning
- The ability to reason about a concept and make connections
- The ability to build on something that has already been mastered
- Conceptual and procedural fluency
Other Features of Mastery

Factual & Procedural Fluency

Conceptual Understanding

INTEGRATION
The 3 Aims of the National Curriculum

- Fluency
- Reasoning
- Problem Solving
3 Forms of Knowledge

Factual – I know that

Procedural – I know how

Conceptual – I know why
What does mastery mean for teaching

- Teachers reinforce an **expectation that all pupils** are capable of achieving high standards in mathematics.
- The large majority of pupils progress through the curriculum content **at the same pace**. Differentiation is achieved by emphasising deep knowledge and through individual support and intervention.
- Teaching is underpinned by **methodical curriculum design** and supported by carefully crafted lessons and resources to foster deep conceptual and procedural knowledge.
- **Practice and consolidation** play a central role. Carefully designed variation within this builds fluency and understanding of underlying mathematical concepts in tandem.
- Teachers use **precise questioning** in class to test conceptual and procedural knowledge, and assess pupils regularly to identify those requiring intervention so that all pupils keep up.
Let's do a bit of Maths

\[ \square + 17 = 15 + 24 \]
\[ 99 - \square = 90 - 59 \]
The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils’ understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on. (NCp3)
What are the benefits and challenges?

- Fewer Things
- Greater Depth
- Class working together
- Longer time on topics

Together these reflect the features of

**A Mastery Curriculum for Mathematics**
The Chinese Paradox

The paradox does not exist because good quality teaching actually takes place in Chinese mathematics Classrooms (Huang 2004)
Deep Conceptual Learning

It is essential to enquire into the origins of the applications of the methods so they will not be forgotten for a long time.

It is difficult to see the logic and method behind complicated problems. Simple problems are hereby given and elucidated. Once these are understood, problems however difficult will become clear (Yang Hui 1274).
## Curriculum Design and Lesson Planning

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Curriculum design</strong></td>
<td>Longer units of work, prioritising key topics</td>
</tr>
<tr>
<td><strong>Lesson design</strong></td>
<td>Carefully structured lesson to develop the detail and depth</td>
</tr>
<tr>
<td><strong>Pupil support</strong></td>
<td>Quick intervention</td>
</tr>
<tr>
<td><strong>Teaching resources</strong></td>
<td>Carefully chosen examples and activities. Application of variation theory. Effective use of representations</td>
</tr>
<tr>
<td><strong>Teaching methods differentiation</strong></td>
<td>Keeping the class together and aiming for depth</td>
</tr>
<tr>
<td><strong>Productivity and practice</strong></td>
<td>Intelligent practice</td>
</tr>
</tbody>
</table>
A Lesson from Shanghai
Features of a Grade 2 Lesson: introduction to Multiplication

- Linking to previous concepts
- Introduction of the correct mathematical language
- Use of context and images – children in boats
- Progression from simple to more complex

Start of the lesson: $3 + 3 + 3 + 3 = 4 \times 3$

End of the lesson $2 + 2 + 2 + 2 + 4 = 6 \times 2$ or $3 \times 4$

$3 + 3 + 3 + 2 + 4 = 5 \times 3$
Features of a Grade 4 Lesson on multiplication

• Use of precise mathematical language
• Describing mathematical relationships
  \[ \text{dividend} = \text{quotient} \times \text{divisor} \]
• Missing number problems
• Challenging thinking and reasoning – which one is the answer?
Teaching the Chinese Way

- Whole class teaching
- Carefully crafted lesson design
- Step by step (scaffolded) logical approach, leading pupils to a deeper understanding
- Constructivist learning
- Providing the opportunity for pupils to engage, discuss and think about mathematics
- Homework – to provide additional practice
The teacher presents a maths problem
And then asks:

1. **What** is the answer?
2. **Describe** the method/procedure you used
3. **Why** does the method work, what relationships are involved, what generalities or rules can we glean?
The implications for assessment

Mastery does not accelerate, instead it deepens.

What applications does this have on assessment of progress?
Practice Makes Perfect

Repetition – can be superficial
Intelligent practice
become **fluent** in the fundamentals of mathematics, including through varied and frequent **practice** with increasingly complex problems over time, *so that pupils develop conceptual understanding* and are able to recall and apply their knowledge rapidly and accurately.
Many Western educators hold the view that students should be encouraged to understand rather than to memorise what they are learning (Purdie, Hattie & Douglas, 1996) as they believe that understanding is more likely to lead to high quality outcomes than memorizing (Dahlin & Watkins, 2000).
Are repetition and understanding in opposition to each other?

Chinese teachers do not see repetition and understanding as separate but rather as interlocking processes, complementary to each other........

We argue that equating repetitive learning with “surface learning without understanding” oversimplifies and misinterprets the intrinsic meaning of the Chinese notion of learning. (Lai et al 2012)
China and Intelligent practice

Tasks that are used:

• Involve reasoning
• Expose the structure of the mathematics
• Help pupils to generalise and apply to “harder mathematics”
• Help pupils to make connections between interrelated mathematical ideas.

These support intelligent practice
Practice makes perfect!

Is doing a large number of exercises compatible with deep learning?

It depends on the nature of the practice – Intelligent practice provides opportunity for The development of procedural fluency and conceptual understanding in tandem Variation theory
Procedural Variation

In designing these exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity (Gu, 1991).
用小圆片摆一摆，试一试。
Procedural Variation – where successive problems link to the previous problem

<table>
<thead>
<tr>
<th>2 × 3 =</th>
<th>6 × 7 =</th>
<th>9 × 8 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 30 =</td>
<td>6 × 70 =</td>
<td>9 × 80 =</td>
</tr>
<tr>
<td>2 × 300 =</td>
<td>6 × 700 =</td>
<td>9 × 800 =</td>
</tr>
<tr>
<td>20 × 3 =</td>
<td>60 × 7 =</td>
<td>90 × 8 =</td>
</tr>
<tr>
<td>200 × 3 =</td>
<td>600 × 7 =</td>
<td>900 × 8 =</td>
</tr>
</tbody>
</table>
Variation supports intelligent practice.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7 + 2 =</td>
<td>9 + 6 =</td>
<td>8 + 3 =</td>
<td>1 + 9 =</td>
</tr>
<tr>
<td>17 + 2 =</td>
<td>10 + 6 =</td>
<td>10 + 3 =</td>
<td>2 + 8 =</td>
</tr>
<tr>
<td>7 + 12 =</td>
<td>11 + 6 =</td>
<td>12 + 3 =</td>
<td>3 + 7 =</td>
</tr>
<tr>
<td>17 + 12 =</td>
<td>13 + 6 =</td>
<td>3 + 14 =</td>
<td>6 + 4 =</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9 − 5 =</td>
<td>9 − 7 =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 − 5 =</td>
<td>11 − 7 =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 − 5 =</td>
<td>13 − 7 =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 − 5 =</td>
<td>15 − 7 =</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conceptual Variation

(1)

或____

或____

或____

(2)

或____

或____

或____

或____
The role of variation

- Encourages a thinking element to practice
- Embeds a deepening element into practice
- Moves from the simple to the more complex – reasoning and making connections along the way, leading to generality
- Results in deep sustainable learning – MASTERY
Reflection on the Use of Textbooks

Provide coherence
A comprehensive resource
Quality activities
In a class, 18 of the children are girls.

A quarter of the children in the class are boys.

Altogether, how many children are there in the class?